Nomenclature

coordinate system for streamwise, wall-normal, and spanwise directions: $x_i = (x_1, x_2, x_3) = (x, y, z)$

mean velocity for streamwise, wall-normal, and spanwise directions: $U_i = (U_1, U_2, U_3) = (U, 0, 0)$

fluctuating velocity for streamwise, wall-normal, and spanwise directions: $u_i = (u_1, u_2, u_3) = (u, v, w)$

mean pressure and fluctuating pressure: P, p

density: ρ

kinetic viscosity: ν

time: t

$D/Dt \equiv \partial/\partial t + U_1 \partial/\partial x_1$,

< >: averaged value over the homogeneous directions (i.e., x- and z-directions) and time

turbulent kinetic energy per unit mass: k,

$$k = \frac{1}{2}\langle u_i u_i \rangle .$$

turbulent energy dissipation rate per unit mass: $\langle \varepsilon \rangle$,

$$\langle \varepsilon \rangle = \nu \left\langle \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} \right\rangle .$$

uu-budget equation:

$$\frac{D\langle uu\rangle}{Dt} = \underbrace{-2\langle uv\rangle \frac{\partial U}{\partial y}}_{P_{11}} + \underbrace{\frac{\partial}{\partial y}\left(-\langle uuv\rangle\right)}_{T_{11}} + \underbrace{2\left\langle \frac{p}{\rho}\frac{\partial u}{\partial x}\right\rangle}_{\phi_{11}} + \underbrace{\frac{\partial}{\partial y}\left(\nu \frac{\partial \langle uu\rangle}{\partial y}\right)}_{V_{11}} \underbrace{-2\nu\left\langle \frac{\partial u}{\partial x_j}\frac{\partial u}{\partial x_j}\right\rangle}_{\varepsilon_{11}} = 0 .$$

vv-budget equation:

$$\frac{D\langle vv\rangle}{Dt} = \underbrace{\frac{\partial}{\partial y}\left(-\langle vvv\rangle\right)}_{T_{22}} + \underbrace{2\frac{\partial}{\partial y}\left(-\left\langle \frac{pv}{\rho}\right\rangle\right)}_{\Pi_{22}} + \underbrace{2\left\langle \frac{p}{\rho}\frac{\partial v}{\partial y}\right\rangle}_{\phi_{22}} + \underbrace{\frac{\partial}{\partial y}\left(\nu \frac{\partial \langle vv\rangle}{\partial y}\right)}_{V_{22}} \underbrace{-2\nu\left\langle \frac{\partial v}{\partial x_j}\frac{\partial v}{\partial x_j}\right\rangle}_{\varepsilon_{22}} = 0 ,$$

$\Pi_{22} + \phi_{22} \equiv PVG$.

ww-budget equation:

$$\frac{D\langle ww\rangle}{Dt} = \underbrace{\frac{\partial}{\partial y}\left(-\langle wwv\rangle\right)}_{T_{33}} + \underbrace{2\left\langle \frac{p}{\rho}\frac{\partial w}{\partial z}\right\rangle}_{\phi_{33}} + \underbrace{\frac{\partial}{\partial y}\left(\nu \frac{\partial \langle ww\rangle}{\partial y}\right)}_{V_{33}} \underbrace{-2\nu\left\langle \frac{\partial w}{\partial x_j}\frac{\partial w}{\partial x_j}\right\rangle}_{\varepsilon_{33}} = 0 .$$

k-budget equation:

$$\frac{Dk}{Dt}=\underbrace{\frac{P_{11}}{2}}_{P_k}+\underbrace{\frac{T_{11}+T_{22}+T_{33}}{2}}_{T_k}+\underbrace{\frac{\Pi_{22}}{2}}_{\Pi_k}+\underbrace{\frac{V_{11}+V_{22}+V_{33}}{2}}_{V_k}-\underbrace{\frac{\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}}{2}}_{\langle\varepsilon\rangle}=0\cdot$$

<u>*uw*-budget equation</u>:

$$\frac{D\langle uv\rangle}{Dt}=\underbrace{-\langle vv\rangle\frac{\partial U}{\partial y}}_{P_{12}}+\underbrace{\frac{\partial}{\partial y}\left(-\langle uvv\rangle\right)}_{T_{12}}\underbrace{-\left\langle\frac{p}{\rho}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\rangle}_{\phi_{12}}\underbrace{-\frac{1}{\rho}\frac{\partial\langle pu\rangle}{\partial y}}_{\Pi_{12}}$$

$$+\underbrace{\frac{\partial}{\partial y}\left(\nu\frac{\partial\overline{uw}}{\partial y}\right)}_{V_{13}}-\underbrace{2\nu\left\langle\frac{\partial u}{\partial x_j}\frac{\partial v}{\partial x_j}\right\rangle}_{\varepsilon_{12}}=0 \quad ,$$

$\Pi_{12}+\phi_{12}\equiv PVG$.

<u>ε-budget equation</u>:

$$\frac{D\varepsilon}{Dt}=\underbrace{-2\nu\left\langle\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}\right\rangle\frac{\partial U}{\partial y}}_{P_\varepsilon^{\,1}}\underbrace{-2\nu\left\langle\frac{\partial u_i}{\partial x}\frac{\partial u_i}{\partial y}\right\rangle\frac{\partial U}{\partial y}}_{P_\varepsilon^{\,2}}\underbrace{-2\nu\left\langle\nu\frac{\partial u}{\partial y}\right\rangle\frac{\partial^2 U}{\partial y^2}}_{P_\varepsilon^{\,3}}\underbrace{-2\nu\left\langle\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_k}\frac{\partial u_k}{\partial x_j}\right\rangle}_{P_\varepsilon^{\,4}}$$

$$+\underbrace{\frac{\partial}{\partial y}\left(-\nu\left\langle\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}v\right\rangle\right)}_{T_\varepsilon}+\underbrace{\frac{\partial}{\partial y}\left(-2\frac{\nu}{\rho}\left\langle\frac{\partial p}{\partial x_i}\frac{\partial v}{\partial x_i}\right\rangle\right)}_{\Pi_\varepsilon} \quad .$$

$$+\underbrace{\nu\frac{\partial^2\langle\varepsilon\rangle}{\partial y^2}}_{D_\varepsilon}-\underbrace{2\nu^2\left\langle\frac{\partial^2 u_i}{\partial x_j\partial x_k}\frac{\partial^2 u_i}{\partial x_j\partial x_k}\right\rangle}_{\gamma}=0$$